

## IMPLICIT HADAMARD FRACTIONAL DIFFERENTIAL EQUATIONS WITH IMPULSES UNDER WEAK TOPOLOGIES

Saïd Abbas<sup>1</sup>, Mouffak Benchohra<sup>2</sup>, John R. Graef<sup>3</sup>, and Jamal-Eddine  
Lazreg<sup>4</sup>

<sup>1</sup>Laboratory of Mathematics  
University of Saïda  
P.O. Box 138, Saïda 20000, Algeria

<sup>2</sup>Laboratory of Mathematics  
Djillali Liabes University of Sidi Bel-Abbes  
P.O. Box 89, Sidi Bel-Abbès 22000, Algeria

<sup>3</sup>Department of Mathematics  
University of Tennessee at Chattanooga  
Chattanooga, TN 37403, USA

<sup>4</sup>Laboratory of Mathematics  
Djillali Liabes University of Sidi Bel-Abbes  
P.O. Box 89, Sidi Bel-Abbès 22000, Algeria

**Abstract.** By applying fixed point theory and the technique of weak measures of non-compactness, the authors prove the existence of weak solutions for some classes of implicit Hadamard fractional differential equations with instantaneous and noninstantaneous impulses.

**Keywords.** Functional differential equation, Pettis Riemann-Liouville integral of fractional order, Hadamard fractional derivative, weak solution, impulses, noninstantaneous impulses, fixed point, measure of weak noncompactness.

**AMS (MOS) subject classification:** 26A33, 34A37

## 1 Introduction

Fractional differential equations have recently been applied in various areas of engineering, mathematics, physics and bio-engineering, and other applied sciences [21, 29]. For some fundamental results in the theory of fractional calculus and fractional differential equations we refer the reader to the monographs of Abbas *et al.* [4, 5], Samko *et al.* [28], Kilbas *et al.* [23] and Zhou [32, 33].

Impulsive differential equations have become more important in recent years in some mathematical models of real phenomena, especially in the biological and medical sciences and in control theory; see, for example, the