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EXISTENCE AND UNIQUENESS RESULTS FOR FRACTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

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Abstract. This paper is devoted to the existence and uniqueness results for fractional differential equations with infinite delay and the Riemann-Liouville fractional derivative order. The technique used to prove our results is by the Banach contraction principle and Schauder's fixed point theorem.

Keywords. Fractional differential equations, Functional differential equations, Fractional derivative and Fractional integral, Existence and uniqueness, Fixed point theorem.

AMS (MOS) subject classification: 34A08, 34K37, 26A33, 34A12, 47H10.

1 Introduction

Recent studies in mechanics, chemistry, engineering, physics, biological sciences and other areas have attracted a considerable interest of researchers and scientists and also these studies have shown that the dynamics of many systems are described more accurately using fractional delay differential equations [2, 3, 6, 7, 8, 10, 17, 18, 19, 23, 25], that are often more realistic to describe natural phenomena than those without delay [5, 9, 11, 12, 24], and the references therein. For more details, see the monographs of Abbas et al. [1], Hale [14], Hino [15], Kilbas et al. [16], and Samko et al. [22], Miller and Ross [20], Podlubny [21]. Many authors have addressed studying these equations with finite delay, for example in [23], Ye et al., investigated the existence and uniqueness of a positive solution for some class of differential equation with fractional order and with finite delay

$$D^{\alpha}[y(t)-y(0)] = y(t)f(t,y_t),\ \ t\in[0,T], \tag{1}$$

$$y(t)=\phi(t), \qquad t\in[-\tau,0], \tag{2}$$

where $0<\alpha<1$, D_0^{α} is the standard Riemann-Liouville fractional derivative, $\phi\in\mathcal{C}$, $\phi(0)=0$, $\mathcal{C}$ is the space of continuous function from $[-\tau,0]$ to $\mathbb{R}^{+}$and $f:[0,T]\times\mathcal{C}\longrightarrow\mathbb{R}^{+}$ $(T>0)$ is continuous.