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SOME REMARKS ON A ONE-DIMENSIONAL LAPLACIAN-LIKE PROBLEM VIA A LOCAL MINIMUM THEOREM

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Abstract. Under an asymptotical behaviour of the nonlinear term, we establish that the problem

$$-\left(\left(1+\frac{{u'}^2}{\sqrt{1+{u'}^4}}\right)u'\right)' = \lambda f(t,u) \quad \text{in } (0,1), \qquad u(0)=u(1)=0,$$

where $\lambda \in \mathbb{R}$ and $f:[0,1]\times\mathbb{R}\to\mathbb{R}$ is a Carathéodory function vanishing at zero, admits at least one non-trivial and non-negative weak solution u.

Keywords. Laplacian-like problem, existence results, critical points, variational method, weak solutions.

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1 Introduction

Capillarity can be briefly explained by considering the effects of two opposing forces: adhesion, that is, the attractive (or repulsive) force between the molecules of the liquid and those of the container; and cohesion, that is, the attractive force between the molecules of the liquid. The study of capillary phenomena has gained some attention recently. This increasing interest is motivated not only by fascination in naturally-occurring phenomena such as motion of drops, bubbles and waves but also its importance in applied fields ranging from industrial and biomedical and pharmaceutical to microfluidic systems. The equation

$$\begin{cases} -\mathrm{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = f(t,u) \quad \text{in } \Omega, \\ u|_{\partial\Omega} = 0 \end{cases} \tag{1}$$

plays, as is well known, a role in differential geometry and in the theory of relativity. Existence, non-existence and multiplicity of positive solutions of problem (1) have been discussed by several authors in the last decades.