Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **26 (2019)** 317-327
 http://www.watam.org

OSCILLATION CRITERIA FOR SECOND ORDER DIFFERENTIAL EQUATIONS WITH A SUB-LINEAR NEUTRAL TERM

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Abstract. This paper deals with oscillation of second order differential equation with a sub-linear neutral term of the form

$$(a(t)(x(t) + p(t)x^{\alpha}(\tau(t)))')' + q(t)x^{\beta}(\sigma(t)) = 0, \ t \geq t_0 \geq 0$$

where $0 < \alpha \leq 1$ and β are ratio of odd positive integer. Using some fundamental inequalities and Ricaati type transformation four new oscillation criteria are obtained. Examples are provided to illustrate the main results.

Keywords. Oscillation, sub-linear neutral term, second order, differential equation.

AMS(MOS) subject classification: 34C10, 34K11.

1 Introduction

In this paper, we investigate the oscillatory behavior of the nonlinear differential equation with a sublinear neutral term

$$(a(t)(z(t))')' + q(t)x^{\beta}(\sigma(t)) = 0, \ t \geq t_0 \geq 0 \tag{1.1}$$

where $z(t) = x(t) + p(t)x^{\alpha}(\tau(t))$, subject to the following conditions:

1. $0 < \alpha \leq 1$ and β are ratios of odd positive integers;
2. $a \in C^1([t_0, \infty), (0, \infty))$, $p, q \in C([t_0, \infty), \mathbb{R})$, $p \geq 0$, and $q \geq 0$ and is not eventually zero on any half line $[t_*, \infty)$ for $t_* \geq t_0$;
3. $\tau \in C([t_0, \infty), \mathbb{R})$, $\sigma \in C^1([t_0, \infty), \mathbb{R})$, $\tau(t) \leq t$, $\sigma(t) \leq t$, $\sigma'(t) > 0$, and $\lim\limits_{t\to\infty} \tau(t) = \lim\limits_{t\to\infty} \sigma(t) = \infty$.