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A GENERAL PIECEWISE SPLINE MAXIMUM ENTROPY METHOD FOR POSITION DEPENDENT RANDOM MAPS

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Let $\{\tau_1, \tau_2, \cdots, \tau_K\}$ be a collection of nonsingular maps on $[0,1]$ into $[0,1]$ and consider a collection $\{p_1(x), p_2(x), \cdots, p_K(x)\}$ of position dependent probabilities on $[0,1]$ into itself. We consider position dependent random maps $T = \{\tau_1, \tau_2, \cdots, \tau_K; p_1(x), p_2(x), \cdots, p_K(x)\}$ such that T preserves a unique absolutely continuous invariant measure μ^* with density f^*. A general piecewise spline maximum entropy method for the approximation of f^* is developed. We present a proof of convergence of the general piecewise spline maximum entropy method for position dependent random maps. We also present numerical examples.

Keywords. Boltzmann Entropy; General Piecewise Spline Maximum Entropy Method; Position Dependent Random Maps; Absolutely Continuous Invariant Measures;

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1 Introduction

A random map T on a set X is a special type of random dynamical system which consists a number of transformations from X into itself where the process switches from one map to another according to fixed probabilities [32] or, more generally, position dependent probabilities. Random maps have application in the study of fractals [3], in modeling interference effects in quantum mechanics [8], in computing metric entropy [35], in forecasting the financial markets (see [33] and Section 7.2 of [34]) and many other areas in science and engineering.

Numerical computation of absolutely continuous invariant measures for position dependent random maps is one of the major research topics in random dynamical systems. Absolutely continuous invariant measures for position dependent random maps reflect their long time behaviour and play an important role in understanding their statistical properties and chaotic nature. Fixed points of Frobenius-Perron operator of a dynamical system are invariant densities of absolutely continuous invariant measures of the system. The Frobenius-Perron equation is a functional equation and it is difficult to solve this equation except in some simple cases. In this paper, we develop