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# MATHEMATICAL FOUNDATIONS OF IDE THEORY

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**Abstract.** This paper presents necessary conditions under which an ODE or IDE can be determined to have the potential for chaotic solutions based on its *form*. In order to do this it is necessary for the algebraic units of stretching and folding to be present in the form of the ODE or IDE. An algebraic element of stretching is an exponential function. An algebraic element of folding is a periodic function or an exponential function. Therefore, a sufficient condition for stretching is the presence of an exponential dynamic. A sufficient condition for folding is the presence of a periodic dynamic or an exponential dynamic.

**Keywords.** chaos, natural science, complexity, dynamical systems dynamical synthesis.

**AMS (MOS) subject classification:** 37D45.

## 1 Introduction

This paper provides sufficient conditions under which the *form* of an ODE or IDE will assure that they have the potential to have chaotic solutions. Section 2 examines the concept of *randomness* and its relationship to the fundamental units of chaos. Section 3 identifies the fundamental unit of chaos that is needed to generate a chaotic sequence.

## 2 Randomness

There exist no formal, mathematical definition of *randomness*. This fact was observed by Kolmogorov, Chaitin, and Solomonov among others. *Random* is an intuitive notion best considered as a metaphor. Thus, randomness cannot be used to *formally* define *chaos*. Ford has explored this concept extensively, [1]. In many cases, the formal mathematical alternative to using the term *Random* in reference to chaos is to designate a Bernoulli automorphism, or shift, as the canonical example of chaos and derive other chaotic systems based on their relationship to the Bernoulli shift [2].

The decision to standardize all chaotic systems around the Bernoulli shift derives from Ergodic Theory in which the Bernoulli shift is recognized as the idealization