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ON A RATIONAL SECOND ORDER DIFFERENCE EQUATION WITH QUADRATIC TERM

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Abstract. In this paper, we solve the difference equation

$$x_{n+1} = \frac{\alpha}{x_n(-1 + x_{n-1})}, \quad n = 0, 1, ...,$$

where $\alpha > 0$ and the initial values x_{-1}, x_0 are real numbers. We give an invariant set of Lebesgue measure zero. We show the existence of periodic solutions and other solutions converge to periodic solutions. Finally, we give some numerical examples.

Keywords. difference equation, forbidden set, convergence, bounded, invariant set.

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1 Introduction

In this paper, we study the forbidden set, solution and the existence of periodic solutions will be studied for the difference equation with quadratic term

$$x_{n+1} = \frac{\alpha}{x_n(-1 + x_{n-1})}, \quad n = 0, 1, ..., \tag{1}$$

where α is a positive real number and the initial conditions are real numbers. For more results on difference equations, see [1–13], [15–19], [21], [24–29], [31–34] and the references therein.
The transformation

$$x_n = \frac{w_{n-1}}{w_n}, \quad \text{with} \quad w_{-2} = 1,$$

reduces the difference equation (1) into the linear third order difference equation

$$w_{n+1} + \frac{1}{\alpha} w_{n-1} - \frac{1}{\alpha} w_{n-2} = 0, \quad n = 0, 1, ... \tag{2}$$

The characteristic equation of equation (2) is

$$\lambda^3 + \frac{1}{\alpha}\lambda - \frac{1}{\alpha} = 0. \tag{3}$$