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UNBOUNDED SOLUTION OF BOUNDARY VALUE PROBLEM FOR NONLINEAR CAPUTO-TYPE ERDELYI KOBER FRACTIONAL DIFFERENTIAL EQUATION ON THE HALF-LINE

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Abstract. In this paper, we investigate the existence and uniqueness of solutions to a class of boundary value problem of nonlinear Caputo-type Erdélyi Kober fractional differential equations on infinite interval. Some new existence and uniqueness results of solutions for the given problem are obtained by means of Schauder's, nonlinear alternative Leray-Schauder's and contraction mapping principle fixed point theorems. Moreover, two examples are presented to illustrate the usefulness of our main results.

Keywords. Fractional differential equation, Boundary value problem, Caputo-type Erdélyi Kober derivative, Fixed point theorem, Existence, Uniqueness.

AMS (MOS) subject classification: 34A08, 34A37, 47H10.

1 Introduction

In this paper, we discuss the existence and uniqueness of the unbounded solution for boundary value problem of nonlinear fractional differential equation involving Caputo-type Erdélyi Kober fractional differential operator on infinite interval

$$ {}_*\mathcal{D}_{\beta}^{\gamma,\delta} u\,(t) + f\,(t, u\,(t)) = 0,\ \ t > 0, \tag{1}$$

with the boundary conditions:

$$\lim_{t\to 0} t^{\beta(1+\gamma)} u^{(k)}(t) = 0,\ \text{ and } \lim_{t\to\infty} t^{\beta(1+\gamma)} u(t) = \rho u\,(1)\,,\ \text{ with } k = \overline{0, n-2}, \tag{2}$$

where ${}_*\mathcal{D}_{\beta}^{\gamma,\delta}$ is the Caputo-type Erdélyi Kober fractional derivative operator of order δ, such that $n-1 < \delta \leq n$, $-n < \gamma < 1-n$, $n \in \mathbb{N}$, $n \geq 2$, $\beta > 0$, $0 < \rho < 1$, and f is a given function required to satisfy certain conditions when we study the existence and uniqueness of the unbounded solution.

The fractional calculus is based on several definitions for the operators of integration and differentiation of arbitrary order. The Caputo-type Erdélyi Kober fractional derivatives are the most useful operators, which introduced