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ON A MATHEMATICAL MODEL OF THERMAL EXPLOSION WITH MULTIPLE PARAMETERS AND NONLINEAR BOUNDARY CONDITIONS

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Abstract. This paper deals with the existence and multiplicity of positive solutions for a mathematical model of thermal explosion with multiple parameters and nonlinear boundary conditions. The arguments rely on the method of sub– and supersolutions.

Keywords. Positive solutions; Thermal explosion; Sub-supersolutions.

AMS (MOS) subject classification:35J55, 35J65.

1 Introduction

A classical problem in combustion theory is a model of thermal explosion which occurs due to a spontaneous ignition in a rapid combustion process. In this paper, we consider a model involving a nonlinear boundary heat loss which is not a very typical one in classical combustion theory, but is relevant to some more applications (see [6, 12, 14, 7] for details). The model reads as:

$$\begin{cases} \theta(t) - \Delta\theta = \alpha_1 f(\eta) + \beta_1 h(\theta), & (t,x) \in (0,\infty)\times\Omega, \\ \eta(t) - \Delta\eta = \alpha_2 g(\theta) + \beta_2 k(\eta), & (t,x) \in (0,\infty)\times\Omega, \\ \mathbf{n}.\nabla\theta + a(\theta)\theta = 0, & (t,x) \in (0,\infty)\times\partial\Omega, \\ \mathbf{n}.\nabla\eta + b(\eta)\eta = 0, & (t,x) \in (0,\infty)\times\partial\Omega, \\ \theta(0,x) = 0 = \eta(0,x). & \end{cases} \tag{1}$$

Here θ, η are the appropriately scaled temperature in a bounded smooth domain $\Omega \subset R^N$, $N \geq 1$, and f, g, h, k are the normalized reactions rate. We assume that f, g, h, k satisfying the following assumptions:

(H1) $f, g, h, k \in C^1(0,\infty) \cap C[0,\infty)$ are strictly increasing functions such that $f(0) > 0$, $g(0) > 0$, $h(0) \geq 0$, $k(0) \geq 0$ and $\lim_{x\to\infty} g(x) = \infty$.

(H2) $\lim_{s\to\infty} \frac{f(Ag(s))}{s} = 0$, for all $A > 0$.

and