ASYMPTOTIC ANALYSIS FOR A NONLINEAR VISCOELASTIC PROBLEM WITH INFINITE HISTORY UNDER A WIDER CLASS OF RELAXATION FUNCTIONS

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Abstract. In this paper, we consider a nonlinear viscoelastic problem with infinite history and a nonlinear feedback localized on the domain and a relaxation function satisfying

$$g'(t) \leq -\xi(t)G(g(t)).$$

We establish explicit and general decay rate results, using the multiplier method and some properties of the convex functions. Our results are obtained without imposing any restrictive growth assumption on the damping term and without imposing any assumption on the boundedness of initial data used in many earlier papers in the literature.

Keywords. General decay, Infinite memory, Relaxation function, Viscoelasticity.

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1 Introduction

In this paper, we consider the following nonlinear viscoelastic problem:

$$\begin{cases} u_{tt} - \Delta u + \int_0^{+\infty} g(s)\Delta u(t-s)ds + \sigma(t)h(u_t) = 0, & \text{in } \Omega \times (0,\infty) \\ u = 0, & \text{on } \partial\Omega \times (0,\infty) \\ u(x,-t) = u_0(x,t), u_t(x,0) = u_1(x), & \text{in } \Omega \times (0,\infty), \end{cases}$$
$$(1)$$

where u denotes the transverse displacement of waves, Ω is a bounded domain of $\mathbb{R}^N(N \geq 1)$ with a smooth boundary $\partial\Omega$ and g, h, σ are specific functions. The viscoelastic problems with infinite-memory terms have been studied by several authors. Giorgi et al. [12] considered the following semilinear hyperbolic equation, in a bounded domain $\Omega \subset \mathbb{R}^3$,

$$u_{tt} - K(0)\Delta u - \int_0^{+\infty} K'(s)\Delta u(t-s)ds + g(u) = f$$