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EXISTENCE AND BLOW-UP RESULT FOR WAVE EQUATION WITH VARIABLE EXPONENT

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Abstract. In the present work, we consider a wave equation with sign changing coefficient and variable exponent nonlinear source term. Using analytical arguments and appropriate estimates of some functionals, we establish the well-posedness and show that the solutions blows up in finite time. This result complements and generalize previous results in the literature.

Keywords. Wave Equation, Nonlinear source, Well-posedness, blow up, sign changing, variable exponent.

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1 Introduction

In this paper, we consider the following problem

$$\begin{cases} u_{tt}(x,t) - \Delta u(x,t) + \delta u_t(x,t) \\ \qquad\qquad +a(x,t)u(x,t) = |u(x,t)|^{p(x)-2}u(x,t), & \text{in } \Omega \times (0,T), \\ u = 0, & \text{on } \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & \text{in } \Omega, \end{cases} \tag{1}$$

where $\delta > 0$, $0 < T < \infty$, $\Omega \subset \mathbb{R}^n (n \geq 2)$ is a bounded domain with smooth boundary $\partial\Omega$, $a = a(x,t)$ is a bounded continuous and sign changing function. For example, if $f : [0,\infty) \to (-\infty,\infty)$ is any function and $g : \Omega \to (-\infty,\infty)$ is a bounded function, then $a(x,t) = (\text{sign}f)(t)g(x)$ is an example of a sign changing function. The variable exponent $p(x)$ is a given measurable function on Ω satisfying

$$2 \leq p_1 \leq p(x) \leq p_2 \leq \frac{2n}{n-2}, \quad n \geq 3, \tag{2}$$

where

$$p_1 = \operatorname{ess}\inf_{x\in\Omega} p(x), \ p_2 = \operatorname{ess}\sup_{x\in\Omega} p(x)$$