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DYNAMICS OF A THIRD ORDER DIFFERENTIAL EQUATION WITH PIECEWISE CONSTANT ARGUMENT OF GENERALIZED TYPE

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Abstract. In this work, we address a third order differential equation with generalized piecewise constant argument. Investigation concerning the existence, uniqueness of the solutions of the equation is performed. Sufficient conditions that guarantee the existence of uniformly asymptotically stable trivial solution are given on the basis of both Lyapunov-Krasovskii and Lyapunov-Razumikhin methods. The results are supported by an example and a simulation.

Keywords. Uniform asymptotic stability, Lyapunov functional, Lyapunov function, piecewise constant argument of generalized type, third order differential equation, simulation.

AMS (MOS) subject classification: 34A36, 34A34, 34K20, 34K60.

1 Introduction and Preliminaries

Differential equations allow to construct real processes mathematically and to examine qualitative behavior of them. It is substantially important to discuss models by the approaches which provide more natural perspectives. Differential equations have been enhanced with different types of deviating arguments which are effective for describing behavior of certain real world processes [4, 16, 18]. Piecewise constant argument (PCA) which is one of them plays a remarkable role in the real life applications [16, 17, 19, 20, 21]. There exist intervals of constant length for the switching moments since the greatest integer function is taken for PCA. In the literature, differential equations with PCA (EPCA) which are basically defined as $x'(t) = f(t, x(t), x([t]))$ were generally examined by reducing them into discrete equations. Another argument which has more general form than PCA was introduced as PCA of generalized type (PCAG) in [3, 4] and developed in the papers [5, 6, 7, 15]. Indeed, differential equations with PCAG (EPCAG) were generally given by

$$x'(t) = f(t, x(t), x(\beta(t))), \tag{1}$$