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BOUNDED AND UNBOUNDED POSITIVE SOLUTIONS FOR SINGULAR ϕ-LAPLACIAN BVPS ON THE HALF-LINE WITH FIRST-ORDER DERIVATIVE DEPENDENCE

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Abstract. In this paper, we provide existence results for positive solutions to the singular ϕ-Laplacian boundary value problem

$$\begin{cases} -(\phi(u'))' = a(t)f(t,u,u'),\ t \in (0,+\infty) \\ u(0) = \lim\limits_{t\to+\infty} u'(t) = 0, \end{cases}$$

where $\phi\colon \mathbb{R} \to \mathbb{R}$ is an increasing homeomorphism such that $\phi(0) = 0$, $a\colon (0,+\infty) \to \mathbb{R}^+$ is a measurable function with $a(t) > 0$ a.e. t in some interval of $(0,+\infty)$ and the nonlinearity $f\colon \mathbb{R}^+ \times (0,+\infty) \times (0,+\infty) \to \mathbb{R}^+$ is continuous and may exhibit singular at the solution and at its derivative.

Keywords. ϕ-Laplacian; Singular BVPs; Unbounded intervals; Positive solution; Fixed point theory in cones.

AMS (MOS) subject classification: 34B15, 34B16, 34B18, 34B40.

1 Introduction and main results

This paper concerns existence of positive solutions to the second order boundary value problem (bvp for short)

$$\begin{cases} -\left(\phi(u')\right)'(t) = a(t)f(t,u(t),u'(t)) \text{ a.e. } t > 0, \\ u(0) = \lim\limits_{t\to+\infty} u'(t) = 0, \end{cases} \tag{1.1}$$

where $\phi\colon \mathbb{R} \to \mathbb{R}$ is an increasing homeomorphism such that $\phi(0) = 0$, $a\colon (0,+\infty) \to \mathbb{R}^+$ is a measurable function with $a(t) > 0$ a.e. t in some interval of $(0,+\infty)$ and the nonlinearity $f\colon \mathbb{R}^+ \times (0,+\infty)^2 \to \mathbb{R}^+$ is continuous and may exhibit singular at $u = 0$ and $u' = 0$.