NOVEL POSITIVE SOLUTIONS FOR A CLASS OF IBVP FOR NONLINEAR PARABOLIC EQUATIONS

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Abstract. In this paper, we investigate an IBVP for a class of nonlinear parabolic equations. A new topological approach is applied to prove existence of non-negative classical solutions. The arguments are based upon a recent theoretical result.

Keywords. Nonlinear parabolic equations, Existence, Fixed point theory, Green's function.

AMS (MOS) subject classification: 35K55, 35K45

1   Introduction and relevance of the topic

In this paper, we investigate the following IBVP

$$
\begin{aligned}
&u_t - \Delta u = f(t, x, u, u_x), \quad t > 0, \quad x \in \mathbf{R}^n, \\
&u(0, x) = u_0(x), \quad x \in \mathbf{R}^n, \\
&\lim_{|x| \to \infty} |u(t, x)| < \infty, \quad t \geq 0,
\end{aligned}
\tag{1}
$$

where $u_x = (u_{x_1}, \ldots, u_{x_n})$, $n \in \mathbf{N}$, and

(H1) $f \in \mathcal{C}([0, \infty) \times \mathbf{R}^{2n+1})$,

$$
\begin{aligned}
0 \quad \leq \quad & h_1(x) + h_2(t) \leq f(t, x, u_1, u_2) + h_3(x) + h_4(t) \\
& + \sum_{j=1}^{k} a_j(t, x)|u_1|^{p_j} + \sum_{j=1}^{m} b_j(t, x)|u_2|^{q_j},
\end{aligned}
$$