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BLOW UP AND DECAY OF SOLUTIONS FOR A KIRCHHOFF-TYPE EQUATION WITH DELAY AND VARIABLE-EXPONENTS

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Abstract. This work deals with a nonlinear Kirchhoff type equation with time delay and variable exponents. Firstly, we prove the blow up of solutions. Later, by applying an integral inequality due to Komornik, we obtain the decay result. Those, improve and extend the blow up and decay estimates in the literature.

Keywords. Blow up, Decay, Delay term, Kirchhoff equation, Variable exponent.

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1 Introduction

In this work, we investigate the following Kirchhoff-type equation

$$\begin{cases} u_{tt} - M\left(\|\nabla u\|^2\right)\Delta u + \mu_1 u_t(x,t)\left|u_t\right|^{m(x)-2}(x,t) \\ +\mu_2 u_t(x,t-\tau)\left|u_t\right|^{m(x)-2}(x,t-\tau) \\ = bu\left|u\right|^{p(x)-2} & \text{in } \Omega\times R^+, \\ u(x,t)=0 & \text{in } \partial\Omega\times[0,\infty), \\ u(x,0)=u_0(x),\ u_t(x,0)=u_1(x) & \text{in } \Omega, \\ u_t(x,t-\tau)=f_0(x,t-\tau) & \text{in } \Omega\times(0,\tau), \end{cases} \tag{1}$$

with delay term. Here, $\Omega\subset R^n$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$. $\tau>0$ is a time delay term, μ_1 is a positive constant, μ_2 is a real number and $b\geq 0$ is a constant. $M(s)$ is a positive C^1-function like $M(s)=1+s^\gamma,\ \gamma>0$. The functions u_0, u_1, f_0 are the initial data to be specified later.

$p(\cdot)$ and $m(\cdot)$ are the variable exponents which given as measurable functions on $\overline{\Omega}$ such that:

$$\begin{aligned} 2 &\leq p^-\leq p(x)\leq p^+\leq p^*, \\ 2 &\leq m^-\leq m(x)\leq m^+\leq m^* \end{aligned} \tag{2}$$