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FRACTIONAL HADAMARD AND FEJÉR-HADAMARD INEQUALITIES ASSOCIATED WITH EXP. $(\alpha, H-M)$-CONVEXITY

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Abstract. Convex functions are very useful in mathematical analysis and optimization theory. In this paper, a new generalized convexity namely exp. $(\alpha, h-m)$-convexity has been utilized to establish the Hermite-Hadamard and the Fejér-Hadamard inequalities for generalized fractional integral operators containing Mittag-Leffler function via a monotonically increasing function. Furthermore, the Hermite-Hadamard and the Fejér-Hadamard inequalities for exp. $(\alpha-m)$-convexity have been discussed. The presented results have some connection with already published results.

Keywords. Convex functions, Exp. $(\alpha, h-m)$-convex functions, Exp. $(\alpha-m)$-convex functions, Hadamard inequality, Fejér-Hadamard inequality, Generalized fractional integral operators, Mittag-Leffler function.

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1 Introduction

A function $f: I \to R$ (where $I \subseteq R$ is an interval) is said to be convex, if for all $\sigma_1, \sigma_2 \in I$ and $\tau \in [0,1]$, the following inequality holds:

$$f(\tau\sigma_1 + (1-\tau)\sigma_2) \leq \tau f(\sigma_1) + (1-\tau) f(\sigma_2). \tag{1}$$

Convex functions are simple in their presentations and are very useful in mathematical analysis, optimization theory and many other subjects of pure and applied nature in science and engineering. Inspired by the inequality (1), researchers get motivation for extending and generalize the notion of convexity. A number of new definitions and concepts have been defined and