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POSITIVE SOLUTIONS FOR A CLASS OF SEMIPOSITONE SINGULAR BOUNDARY VALUE PROBLEMS ON TIME SCALES

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Abstract. In this paper, we consider the existence of multiple positive solutions for the following singular semipositone boundary value problem (BVP):

$$-(\psi(t)y^{\Delta})^{\nabla}(t) = q(t)\,f(t,\ y(t)) + r(t), \quad t \in (\rho(c),\, \sigma(d))_{\mathbb{T}},$$

with mixed boundary conditions

$$\alpha y(\rho(c)) - \beta\psi(\rho(c))y^{\Delta}(\rho(c)) = 0,$$
$$\gamma y(\sigma(d)) + \delta\psi(d)y^{\Delta}(d) = 0,$$

where $\psi : [\rho(c),\, \sigma(d)]_{\mathbb{T}} \to (0, \infty)$ and $f : [\rho(c),\, \sigma(d)]_{\mathbb{T}} \times [0,\, \infty) \to [0,\, \infty)$ are continuous and $q : (\rho(c),\, \sigma(d))_{\mathbb{T}} \to (0,\, \infty)$ is continuous and $r : (\rho(c),\, \sigma(d))_{\mathbb{T}} \to (-\infty,\, \infty)$ Lebesgue ∇-integrable. We establish some sufficient conditions for the existence of multiple positive solutions when f is either sublinear or superlinear by constructing a unique cone and applying a fixed point theorem on this cone. The results obtained herein generalize and improve some known results including singular and non-singular cases. We unify and generalize semipositone results in the continuous and discrete situations to quantum and arbitrary time scales. Examples are also given to demonstrate the significance of the results.

Keywords. Positive solution, Green's function, singular boundary value problems, semipositone, cone, fixed point theorem.

AMS (MOS) subject classification: 34N05, 34B15, 34B16, 34B18, 39A10, 39A13.

1 Introduction

The study of dynamic equations on time scales can be referred to Stefan Hilger's foundational work [17], and it has received a considerable attention in recent years. Time scales were developed to bring together the study of continuous and discrete mathematics, and they are particularly useful in differential and difference equations. Many results concerning differential equations are easily transferred to corresponding difference equations, while others seem to be entirely different from their continuous counterparts. Such inconsistencies can be seen by investigating dynamic equations on time

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